Let G be a td group and (π, V) be a smooth G-representation. Fix a (left) Haar measure μ of G and let $C_c^\infty(G) \simeq \mathcal{H}(G)$ denote the Hecke algebra of G.

1. Let $T \simeq (F^\times)^2$ denote the subgroup of $\mathrm{GL}_2(F)$ consisting of diagonal matrices. Verify that $\mathrm{ind}_T^{\mathrm{GL}_2(F)}(\mathbf{1})$ is NOT admissible.

2. Suppose that $H \subset G$ is closed AND open subgroup and let (σ, W) denote a smooth H-representation. We introduced a canonical H-equivariant map

$$\iota : W \longrightarrow \mathrm{ind}_H^G(W)$$
$$w \longmapsto [g \mapsto \begin{cases} \sigma(g)w & : g \in H, \\ 0 & : \text{otherwise}. \end{cases}]$$

For any smooth G-representation (π, V), prove that this induces a bijection

$$\mathrm{Hom}_G(\mathrm{ind}_H^G(W), V) \simeq \mathrm{Hom}_H(W, V|_H).$$

3. Suppose $H \subset G$ is closed and (σ, W) is a smooth H-representation. Suppose further that $\mathrm{ind}_H^G(W)$ is admissible. Show that

$$\mathrm{ind}_H^G(W) = \mathrm{Ind}_H^G(W).$$

End of assignment